Available online at www.sciencedirect.com

SCIENCE DIRECT®

Applied Mathematics Letters 18 (2005) 395–399

Applied Mathematics Letters

www.elsevier.com/locate/aml

A suggested approach for possibility and necessity dominance indices in stochastic fuzzy linear programming

Maged George Iskander

Department of Economics, The American University in Cairo, P.O. Box 2511, Cairo, Egypt

Received 1 January 2004; accepted 1 April 2004

Abstract

This paper presents a suggested approach for solving a stochastic fuzzy linear programming problem. This approach utilizes two possibility and two necessity dominance indices that have been introduced by Dubois and Prade [D. Dubois, H. Prade, Ranking fuzzy numbers in the setting of possibility theory, Information Sciences 30 (1983) 183–224]. The chance-constrained approach and the α-cut are used to transform the stochastic fuzzy problem to its deterministic-crisp equivalent, according to each of the four dominance indices. A numerical example is given.

Keywords: Possibility of dominance; Necessity of dominance; Chance-constrained approach

1. Introduction

Comparison of fuzzy numbers is considered one of the most important topics in fuzzy logic theory. The early and most important work in the field of comparing fuzzy numbers has been presented by Dubois and Prade [1]. A comparison between their work and other attempts that have been made in this area has been given by Bortolan and Degani [2]. On the other hand, the dominance possibility indices, which have been introduced by Dubois and Prade, were utilized in the field of fuzzy mathematical programming [3,4] and the field of stochastic fuzzy mathematical programming [5,6]. The approach used in these fields was based on formulating a possibility function, whether in the case of trapezoidal

E-mail address: magedgi@aucegypt.edu.

0893-9659/$ - see front matter
doi:10.1016/j.aml.2004.04.010

fuzzy numbers or the case of triangular fuzzy numbers. In this paper, we are going to utilize Dubois and Prade's dominance possibility and necessity indices, within a different approach, in the case of stochastic fuzzy linear programming problem. The dominance possibility and necessity, as well as the strict dominance possibility and necessity criteria, are utilized according to the chance-constrained method to transform the suggested problem to its deterministic-crisp equivalent. This approach helps avoiding any approximation that may exist due to comparing the inverse distribution function of fuzzy tolerance measures.

2. Model specification

In general, consider a stochastic fuzzy linear programming problem of the following form:

$$\text{Maximize} \qquad \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j \tag{1}$$

subject to:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq b_i, \qquad i = 1, \ldots, m, \tag{2}$$

$$x_j \geq 0, \qquad j = 1, \ldots, n. \tag{3}$$

Here x_j, $j = 1, \ldots, n$ are non-negative decision variables, $\tilde{c}_j$, $j = 1, \ldots, n$ are fuzzy coefficients in the objective function, b_i, $i = 1, \ldots, m$ are independent random variables with known distribution functions, while $\tilde{a}_{ij}$ represents the fuzzy coefficient of the jth decision variable in the ith stochastic constraint. Thus, by incorporating fuzzy tolerance measures $\tilde{\delta}_i$, $0 \leq \tilde{\delta}_i \leq 1$, $i = 1, \ldots, m$, and by utilizing the chance-constrained approach, the stochastic fuzzy constraints (2) can be transformed to their deterministic fuzzy equivalents as follows [5,6].

$$\Pr\left(\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq b_i\right) \geq \tilde{\delta}_i, \qquad i = 1, \ldots, m, \tag{4}$$

then,

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq F_i^{-1}(\tilde{\beta}_i), \qquad i = 1, \ldots, m, \tag{5}$$

where $\tilde{\beta}_i = 1 - \tilde{\delta}_i$, and $F_i^{-1}(.)$ is the inverse distribution function of the random variable b_i, $i = 1, \ldots, m$. It is apparent that this transformation requires the independent random variables to be continuous [6–8]. On the other hand, the deterministic fuzzy constraints set (5) is going to be represented by its crisp equivalent, according to each of the following four dominance indices that have been presented by Dubois and Prade [1]: Possibility of Dominance (PD), Possibility of Strict Dominance (PSD), Necessity of Dominance (ND), and Necessity of Strict Dominance (NSD). These indices for comparing fuzzy numbers are utilized whether $\tilde{a}_{ij}$ and $\tilde{\delta}_i$ are presented as trapezoidal or triangular fuzzy numbers.

3. The equivalent deterministic-crisp linear programming models

In this section, the fuzzy objective function (1) as well as the set of deterministic fuzzy constraints (5) are transformed to their crisp equivalents. This transformation is applicable for different types of fuzzy numbers (trapezoidal or triangular).

3.1. The equivalent crisp objective function

Assume that the fuzzy coefficients are considered trapezoidal fuzzy numbers, i.e., $\tilde{c}_j = (\underline{c}_j, c_{j1}, c_{j2}, \overline{c}_j)$. By utilizing the α-cut approach, where α is any predetermined value, $\alpha \in (0, 1]$, then according to Iskander [4], the crisp objective function that is equivalent to (1) can be presented as

$$\text{Maximize } Z = \sum_{j=1}^{n} \left((1-\alpha)\overline{c}_j + \alpha c_{j2}\right) x_j, \tag{6}$$

subject to (5) and (3).

3.2. The equivalent crisp constraints

The crisp equivalent to (5) should be formulated according to each dominance index. The suggested approach, for formulating this set of crisp constraints, depends on utilizing the α-cut approach, for the membership functions of $\tilde{a}_{ij}$ and $\tilde{\beta}_i$. Assuming that $\tilde{a}_{ij}$ and $\tilde{\beta}_i$ are presented as trapezoidal fuzzy numbers, i.e., $\tilde{a}_{ij} = (\underline{a}_{ij}, a_{ij1}, a_{ij2}, \overline{a}_{ij})$ and $\tilde{\beta}_i = (\underline{\beta}_i, \beta_{i1}, \beta_{i2}, \overline{\beta}_i) = (1-\overline{\delta}_i, 1-\delta_{i2}, 1-\delta_{i1}, 1-\underline{\delta}_i)$, then the α-cut for each of the membership functions of $\tilde{a}_{ij}$ and $\tilde{\beta}_i$ should, respectively, derivate the following two closed crisp intervals, $[(1-\alpha)\underline{a}_{ij} + \alpha a_{ij1}, (1-\alpha)\overline{a}_{ij} + \alpha a_{ij2}]$ and $[(1-\alpha)\underline{\beta}_i + \alpha\beta_{i1}, (1-\alpha)\overline{\beta}_i + \alpha\beta_{i2}]$. And since the decision variables are non-negative, then the set of deterministic fuzzy constraints (5) can be transformed to its deterministic-crisp equivalent, according to each of the four dominance indices, which is based on comparing closed crisp intervals [1], as follows:

(a) *According to PD:*

$$\sum_{j=1}^{n}((1-\alpha)\underline{a}_{ij} + \alpha a_{ij1})x_j \le F_i^{-1}((1-\alpha)\overline{\beta}_i + \alpha\beta_{i2}), \qquad i = 1, \ldots, m, \tag{7}$$

(b) *According to PSD:*

$$\sum_{j=1}^{n}((1-\alpha)\overline{a}_{ij} + \alpha a_{ij2})x_j \le F_i^{-1}((1-\alpha)\overline{\beta}_i + \alpha\beta_{i2}), \qquad i = 1, \ldots, m, \tag{8}$$

(c) *According to ND:*

$$\sum_{j=1}^{n}((1-\alpha)\underline{a}_{ij} + \alpha a_{ij1})x_j \le F_i^{-1}((1-\alpha)\underline{\beta}_i + \alpha\beta_{i1}), \qquad i = 1, \ldots, m, \tag{9}$$

(d) *According to NSD:*

$$\sum_{j=1}^{n}((1-\alpha)\overline{a}_{ij} + \alpha a_{ij2})x_j \le F_i^{-1}((1-\alpha)\underline{\beta}_i + \alpha\beta_{i1}), \qquad i = 1, \ldots, m. \tag{10}$$

Table 1
Results of the four dominance indices

α	PD	PSD	ND	NSD
0.3	$x_1 = 0, x_2 = 3.576$ $Z = 31.114$	$x_1 = 2.209, x_2 = 0$ $Z = 20.990$	$x_1 = 0, x_2 = 2.703$ $Z = 23.519$	$x_1 = 1.670, x_2 = 0$ $Z = 15.866$
0.5	$x_1 = 0, x_2 = 3.154$ $Z = 26.808$	$x_1 = 2.216, x_2 = 0$ $Z = 18.838$	$x_1 = 0, x_2 = 2.500$ $Z = 21.250$	$x_1 = 1.757, x_2 = 0$ $Z = 14.932$
0.8	$x_1 = 0, x_2 = 2.649$ $Z = 21.719$	$x_1 = 2.227, x_2 = 0$ $Z = 15.591$	$x_1 = 0, x_2 = 2.257$ $Z = 18.505$	$x_1 = 1.898, x_2 = 0$ $Z = 13.284$

Also, the sensitivity of the results can be tested for other values of α.

It should be noticed that, according to this approach for comparing closed crisp intervals, we can avoid the approximation that may exist due to comparing the inverse distribution function of trapezoidal or triangular fuzzy tolerance measures. This approximation has been presented by using dominance possibility functions for making such comparisons [6]. According to the concept of extension principle, the inverse distribution function of trapezoidal or triangular fuzzy tolerance measures is not exactly trapezoidal or triangular, respectively, except the case of the uniform distribution. Thus, for the other distributions, comparing the inverse distribution function of trapezoidal or triangular fuzzy tolerance measures by utilizing dominance possibility functions, for comparing traditional trapezoidal or triangular fuzzy numbers, is an approximation, although it forms an ideal compromise [3].

On the other hand, the suggested approach can be applied in the case of triangular fuzzy numbers for $\tilde{c}_j, \tilde{a}_{ij}$, and $\tilde{\beta}_i$, i.e., $\tilde{c}_j = (\underline{c}_j, c_{j0}, \overline{c}_j), \tilde{a}_{ij} = (\underline{a}_{ij}, a_{ij0}, \overline{a}_{ij})$, and $\tilde{\beta}_i = (\underline{\beta}_i, \beta_{i0}, \overline{\beta}_i) = (1 - \overline{\delta}_i, 1 - \delta_{i0}, 1 - \underline{\delta}_i)$. In this case, c_{j2} should be replaced by c_{j0} in (6), while a_{ij1} and a_{ij2} should be replaced by a_{ij0}, and also, β_{i0} should replace β_{i1} and β_{i2} in (7)–(10).

It is obvious that a general comparison between the four dominance indices, according to the value of the objective function (6), shows that $Z_{\mathrm{NSD}} \le Z_{\mathrm{PSD}} \le Z_{\mathrm{PD}}$ and $Z_{\mathrm{NSD}} \le Z_{\mathrm{ND}} \le Z_{\mathrm{PD}}$, where $Z_{\mathrm{PD}}, Z_{\mathrm{PSD}}, Z_{\mathrm{ND}}$, and Z_{NSD} are the values of the objective function according to PD, PSD, ND, and NSD, respectively. A numerical comparison is given by the following example.

4. Numerical example

Let the stochastic fuzzy linear programming problem (1)–(3) be presented as follows:

$$\begin{aligned} \text{Maximize}\quad & \tilde{Z} = (2, 5, 6, 11)x_1 + (4, 7, 8, 9)x_2 \\ \text{subject to}\quad & (12, 15, 17, 20)x_1 + (10, 16, 20, 25)x_2 \le b_1, \\ & (0.04, 0.07, 0.08)x_1 + (0.03, 0.06, 0.1)x_2 \le b_2, \\ & x_1, x_2 \ge 0. \end{aligned}$$

Here b_1 and b_2 are independent random variables, with b_1 having a uniform distribution on the interval [30, 50], while b_2 is exponentially distributed with mean equals one. As well, let $\tilde{\delta}_1 = (0.3, 0.6, 0.8, 0.95)$ and $\tilde{\delta}_2 = (0.5, 0.7, 0.9)$. Then, for each of the four dominance indices, the equivalent deterministic-crisp linear programming problem is solved, using the GAMS package [9], when $\alpha = 0.3, 0.5$, and 0.8. The results are presented in Table 1.

5. Conclusion

The suggested approach for comparing fuzzy numbers in the case of stochastic fuzzy linear programming problems can be applied for different types of fuzzy numbers, in addition to the trapezoidal and triangular fuzzy numbers that have been used in this paper. Also, any approximation that may exist, due to using another approach, can be avoided. Utilizing the α-cut technique for the membership functions to derive closed crisp intervals represents the main step in our approach. Thus, for different values of α, and by comparing the closed crisp intervals, results are generated according to each of the four dominance indices, whereas the most convenient one can be chosen.

References

[1] D. Dubois, H. Prade, Ranking fuzzy numbers in the setting of possibility theory, Information Sciences 30 (1983) 183–224.
[2] G. Bortolan, R. Degani, A review of some methods for ranking fuzzy subsets, Fuzzy Sets and Systems 15 (1985) 1–19.
[3] D.S. Negi, E.S. Lee, Possibility programming by the comparison of fuzzy numbers, Computers and Mathematics with Applications 25 (1993) 43–50.
[4] M.G. Iskander, Comparison of fuzzy numbers using possibility programming: comments and new concepts, Computers and Mathematics with Applications 43 (2002) 833–840.
[5] M.Z. El-Asharm, N.M. Girgis, Linear multiobjective programming under randomness and fuzziness, The Journal of Fuzzy Mathematics 4 (1996) 263–276.
[6] M.G. Iskander, Using different dominance criteria in stochastic fuzzy linear multiobjective programming: a case of fuzzy weighted objective function, Mathematical and Computer Modelling 37 (2003) 167–176.
[7] M.G. Iskander, On solving stochastic fuzzy goal programming problem using a suggested interactive approach, The Journal of Fuzzy Mathematics 9 (2001) 437–446.
[8] M.G. Iskander, A fuzzy weighted additive approach for stochastic fuzzy goal programming, Applied Mathematics and Computation 154 (2004) 543–553.
[9] A. Brooke, D. Kendrick, A. Meeraus, R. Raman, GAMS: A User's Guide, GAMS Development Corporation, Washington, DC, 1998.